1D Shock Response Module

This document describes the 1D Shock Response module, its intended use, governing equations, numerical solution method, and relevant references.

1. Module Overview

The 1D Shock Response module models the dynamic response of a suspended weight attached to a linear spring and viscous damper, where the support (base) is subjected to a prescribed acceleration input. The system is a single-degree-of-freedom (SDOF) relative-motion model, commonly used for shock and vibration analysis of mounted equipment.

The base is assumed to be rigid and of effectively infinite mass relative to the suspended weight. The response quantities of interest are the base acceleration and the acceleration experienced by the suspended weight.

2. Physical Model

The system consists of:
- Weight W (mass $m = W / g$)
- Linear spring with stiffness k
- Linear viscous damper with damping coefficient c
- Prescribed base acceleration $a_b(t)$

The relative displacement $z(t) = x(t) − y(t)$ is defined as the displacement of the weight relative to the base. The governing equation of motion is:

$$m \ddot{z} + c \dot{z} + k z = -m a_b(t)$$

3. Inputs

User inputs include:
- Weight, W
- Spring stiffness, k
- Damping coefficient, c
- Gravitational constant, g
- Shock shape (half-sine, triangular, rectangular, sawtooth, or random)
- Peak shock level G_{pk} (for deterministic shocks)
- Pulse duration T_p (for deterministic shocks)
- Integration time step Δt

For the Random option, the user provides a Power Spectral Density (PSD) defined by 2–5 points of acceleration PSD (g^2/Hz) versus frequency (Hz).

4. Derived Quantities

From the inputs, the module computes:
- Natural frequency: $f_n = (1 / 2\pi) \sqrt{k / m}$
- Critical damping: $c_c = 2 m \omega_n$
- Damping ratio: $\zeta = c / c_c$
- Simulation duration based on a fixed number of response cycles

5. Load Definitions

Deterministic shock loads are applied as a single pulse of specified shape and duration, followed by free vibration. Random loads are generated from the user-specified PSD using a random-phase sum-of-sinusoids technique, producing a base acceleration time history.

6. Numerical Solution Method

The equations of motion are solved using a classical fourth-order Runge–Kutta (RK4) time integration scheme. The state vector consists of relative displacement and velocity of the suspended mass, along with base motion states used for animation consistency.

The simulation advances in fixed time steps Δt. Acceleration responses are computed directly from the state derivatives and recorded for plotting.

7. Outputs

The module provides:
- Time history plots of base acceleration and suspended weight acceleration
- Maximum base and weight acceleration levels
- Animated schematic showing relative motion between the weight and the base

8. Usage Notes

For Random excitation, the user must first build the random time history before running the simulation. Changing the PSD requires rebuilding the random input. The animation speed control affects visualization only and does not alter numerical results.

9. Validation Considerations

For undamped systems ($c = 0$), the response should reduce to classical free vibration superimposed on the applied base acceleration. Energy consistency and frequency content should be verified against analytical solutions for simple cases.

10. References

1. Harris, C. M., and Piersol, A. G., Harris' Shock and Vibration Handbook.
2. Clough, R. W., and Penzien, J., Dynamics of Structures.
3. Newmark, N. M., "A Method of Computation for Structural Dynamics," ASCE.
4. Bendat, J. S., and Piersol, A. G., Random Data: Analysis and Measurement Procedures.
5. MIL-STD-810, Environmental Engineering Considerations and Laboratory Tests.