3D Frame Modal Analysis (Modes 1–3)

1. Executive Overview

The 3D Frame Modal Analysis module is a finite element–based structural dynamics application for computing natural frequencies and mode shapes of general 3D space frame structures. The formulation employs six degrees of freedom per node and full 12×12 beam element matrices. The implementation supports arbitrary element orientation, consistent mass formulation, and generalized eigenvalue solution of the dynamic equilibrium equations.

2. User Interface Architecture

2.1 Data Input Panels

Nodes are defined using:
node, x, y, z, delX, delY, delZ, thetaXX, thetaYY, thetaZZ, W

Segments are defined using:
elem, ni, nj, E, G, Izz, Iyy, Jyz, Ayz, rho, x3, y3, z3

Blank entries indicate free DOFs. Prescribed DOFs are enforced via static condensation prior to modal extraction.

2.2 Visualization Engine

The rendering engine provides orbit, pan, zoom, node and element selection, local axis visualization ($\hat{x}$, $\hat{y}$, $\hat{z}$), and animated modal deformation display. Selected nodes display modal displacement vectors and rotations.

2.3 Modal Viewer Controls

After solving, the first three eigenpairs are presented. Amplitude scaling and harmonic animation visualize deformation as:

$$u(t) = \varphi \sin(\omega t)$$

3. Theoretical Formulation

3.1 Governing Eigenvalue Problem

The undamped free vibration equation is:

$$[K]\{\varphi\} = \lambda[M]\{\varphi\}$$

where $\lambda = \omega^2$. Natural frequency is computed as:

$$f = \omega / (2\pi)$$

3.2 Full 12×12 Space Frame Element Stiffness Matrix

The Euler–Bernoulli space frame local stiffness matrix is:

$$
[\,k\,] =
\begin{vmatrix}
EA/L & 0 & 0 & 0 & 0 & 0 & -EA/L & 0 & 0 & 0 & 0 & 0 \\
0 & 12EIzz/L^3 & 0 & 0 & 0 & 6EIzz/L^2 & 0 & -12EIzz/L^3 & 0 & 0 & 0 & 6EIzz/L^2 \\
0 & 0 & 12EIyy/L^3 & 0 & -6EIyy/L^2 & 0 & 0 & 0 & -12EIyy/L^3 & 0 & -6EIyy/L^2 & 0 \\
0 & 0 & 0 & GJ/L & 0 & 0 & 0 & 0 & 0 & -GJ/L & 0 & 0 \\
0 & 0 & -6EIyy/L^2 & 0 & 4EIyy/L & 0 & 0 & 0 & 6EIyy/L^2 & 0 & 2EIyy/L & 0 \\
0 & 6EIzz/L^2 & 0 & 0 & 0 & 4EIzz/L & 0 & -6EIzz/L^2 & 0 & 0 & 0 & 2EIzz/L \\
-EA/L & 0 & 0 & 0 & 0 & 0 & EA/L & 0 & 0 & 0 & 0 & 0 \\
0 & -12EIzz/L^3 & 0 & 0 & 0 & -6EIzz/L^2 & 0 & 12EIzz/L^3 & 0 & 0 & 0 & -6EIzz/L^2 \\
0 & 0 & -12EIyy/L^3 & 0 & 6EIyy/L^2 & 0 & 0 & 0 & 12EIyy/L^3 & 0 & 6EIyy/L^2 & 0 \\
0 & 0 & 0 & -GJ/L & 0 & 0 & 0 & 0 & 0 & GJ/L & 0 & 0 \\
0 & 0 & -6EIyy/L^2 & 0 & 2EIyy/L & 0 & 0 & 0 & 6EIyy/L^2 & 0 & 4EIyy/L & 0 \\
0 & 6EIzz/L^2 & 0 & 0 & 0 & 2EIzz/L & 0 & -6EIzz/L^2 & 0 & 0 & 0 & 4EIzz/L
\end{vmatrix}
$$

3.3 Full 12×12 Consistent Mass Matrix

The consistent mass matrix includes axial, torsional, and bending contributions.

$m = \rho A L$

Axial/Torsional terms:

$$
\begin{vmatrix}
2m/6 & 0 & 0 & 0 & 0 & 0 & 1m/6 & 0 & 0 & 0 & 0 & 0
\end{vmatrix}
$$

| symmetric terms |

Bending submatrix (local z):

$$
(m/420) *
\begin{vmatrix}
156 & 22L & 54 & -13L \\
22L & 4L^2 & 13L & -3L^2 \\
54 & 13L & 156 & -22L \\
-13L & -3L^2 & -22L & 4L^2
\end{vmatrix}
$$

Equivalent form applies about local y.

3.4 Coordinate Transformation

Local-to-global transformation is performed using a 3×3 rotation matrix R constructed from the element axis and orientation vector. The 12×12 transformation matrix is block-diagonal.

3.5 Assembly and Boundary Conditions

Global matrices are assembled via finite element superposition. Prescribed DOFs are removed through static condensation before eigenvalue solution.

4. Validation & Benchmark Cases

4.1 Cantilever Beam (Analytical Comparison)

E = 12000000, I = 10.7, A = 8, ρ = 0.03, L = 96

Analytical frequency:
f_1 = 1.4043 Hz

Mode	Analytical (Hz)	Model Result (Hz)
1	1.4043	User Computed Value

4.2 Fixed–Fixed Beam

Frequency parameter ≈ 4.730^2. Numerical results should converge under mesh refinement.

4.3 Portal Frame

Expected modal behavior:
- Mode 1: Lateral sway
- Mode 2: Beam bending
- Mode 3: Higher-order coupled bending

4.4 Frequency Scaling Verification

- Multiply E by α → frequencies scale by $\sqrt{\alpha}$
- Multiply ρ by β → frequencies scale by $1/\sqrt{\beta}$
- Scale geometry by s → bending frequencies scale $\sim 1/s^2$

5. Numerical Considerations

Euler–Bernoulli assumptions neglect shear deformation. Zero-length elements or collinear orientation vectors produce singularities. Improper constraints may result in rigid-body modes.

6. References

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Cook, R.D. et al., Concepts and Applications of Finite Element Analysis.
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Timoshenko & Young, Vibration Problems in Engineering.