Base Shake Module — Solution Method Description

1. Overview

The Base Shake module models the vertical and rocking response of a rigid mass supported by four corner springs on a base that is subjected to a prescribed vertical base acceleration ("base shake"). The top body is constrained to move as a rigid block with one vertical translation and two small rotations about the horizontal axes. The base itself is modeled as a lumped mass connected to ground by a vertical spring–damper pair. The module computes the transient response using an adaptive Runge–Kutta–Fehlberg (RKF45) time integrator and outputs acceleration at the top center and reaction forces in the corner springs.

2. Geometry, Degrees of Freedom, and Coordinates

The model assumes a rigid top block of plan dimensions Lx (width) and Ly (length), and height Lz. The four vertical support springs are located at the plan corners:

- • Corner 0: (0, 0)
- • Corner 1: (Lx, 0)
- • Corner 2: (0, Ly)
- • Corner 3: (Lx, Ly)

For dynamic calculations, the code defines corner coordinates relative to the top block center of mass (COM). If the COM is at (Lx/2, Ly/2), then the local corner coordinates relative to the COM are:

- • Corner 0: (−Lx/2, −Ly/2)
- • Corner 1: (+Lx/2, −Ly/2)
- • Corner 2: (−Lx/2, +Ly/2)
- • Corner 3: (+Lx/2, +Ly/2)

The degrees of freedom (DOFs) are defined as follows:

- • z — vertical translation of the top COM
- • θx — rotation of the top about the global x-axis (roll)
- • θy — rotation of the top about the global y-axis (pitch)
- • vz — vertical velocity of the top COM
- • ωx — angular velocity about x
- • ωy — angular velocity about y
- • zb — vertical displacement of the base mass
- • vb — vertical velocity of the base mass

The state vector is therefore:

$y = [\, z, \theta x, \theta y, vz, \omega x, \omega y, zb, vb \,]^T$

3. Mass and Inertia Properties

The user specifies the weight W of the top block. The code converts weight to mass using the selected gravity constant g (depending on US or SI units):

$$m\_top = W / g$$

The mass moments of inertia of the rigid top block about its principal axes through the COM are approximated as those of a homogeneous rectangular block:

$$I\_x = (m\_top / 12) \cdot (Ly^2 + Lz^2)$$

$$I\_y = (m\_top / 12) \cdot (Lx^2 + Lz^2)$$

The base is modeled as a separate lumped mass m_base, chosen as a multiple of the top mass (m_base = 100 · m_top in the code) to represent a relatively heavy base structure. This mass is attached to ground by a vertical spring–damper pair.

4. Corner Springs and Damping

Each of the four support locations has a vertical spring with user-defined stiffness values k_{00}, k_Lx0, k_0Ly, and k_LxLy. To compute the vertical displacement and velocity at each corner of the top, the code uses small-angle rigid-body kinematics. For corner i with COM-relative coordinates (x_i, y_i), the vertical displacement of the top surface at that corner is:

$$u\_top,i = z - \theta x \cdot y_i + \theta y \cdot x_i$$

and the corresponding vertical velocity is:

$$v\_top,i = vz - \omega x \cdot y_i + \omega y \cdot x_i$$

The base displacement and velocity are zb and vb. Therefore the relative motion across spring i is:

$$\Delta u_i = u\_top,i - zb$$

$$\Delta v_i = v\_top,i - vb$$

Each spring is modeled as a linear spring–dashpot element with viscous damping. The code derives a damping coefficient for each corner based on a user-specified fraction of critical damping ζ ("Critical damping, %"). The equivalent modal mass per corner is taken as m_eq = m_top / 4, and the corner damping coefficients are:

$$c_i = 2 \zeta \sqrt{(k_i \cdot m\_eq)}$$

The force in corner spring i, acting on the top mass, is then defined as:

$$F_i = - (k_i \Delta u_i + c_i \Delta v_i)$$

5. Base Support to Ground

The base mass is connected to ground by a vertical spring and damper. The base spring stiffness k_ground is taken as a fraction of the average corner stiffness:

$$k\_ground = 0.1 \cdot (k_{00} + k\_Lx0 + k\_0Ly + k\_LxLy) / 4$$

The corresponding viscous damping coefficient for the base support is defined using the same damping ratio ζ:

$$c\_ground = 2 \zeta \sqrt{(k\_ground \cdot m\_base)}$$

The restoring force transmitted between the base mass and ground is:

$$F\_ground = - k\_ground \cdot zb - c\_ground \cdot vb$$

6. Governing Equations of Motion

The governing equations are derived by summing forces and moments on the top block and on the base mass. For the top, the net vertical force F_z and the net roll and pitch moments τ_x and τ_y are obtained from the corner forces:

$$F\_z = \Sigma F_i$$

$$\tau\_x = \Sigma (-y_i \cdot F_i)$$

$$\tau\_y = \Sigma (x_i \cdot F_i)$$

Using these, the translational and rotational accelerations of the top are:

$$\ddot{z} = F\_z / m\_top$$

$$\ddot{\theta}x = \tau\_x / I\_x$$

$$\ddot{\theta}y = \tau\_y / I\_y$$

For the base mass, the vertical equation of motion includes the imposed base acceleration a_base(t) as well as interaction with the top and ground. In the implementation, the base acceleration in model units is:

$$a\_base(t) = a\_base,G(t) \cdot g$$

where a_base,G(t) is specified in units of G (multiples of gravity). The base acceleration equation is then written in terms of the base mass acceleration $\ddot{z}\_b$:

$$m\_base \cdot \ddot{z}\_b = m\_base \cdot a\_base(t) + (-F\_z + F\_ground)$$

Rearranged for numerical integration, the state derivatives used in the code are:

$$\dot{z} = vz$$

$\dot{\theta}x = \omega x$

$\dot{\theta}y = \omega y$

$\dot{v}z = \ddot{z} = F\_z / m\_top$

$\dot{\omega}x = \ddot{\theta}x = \tau\_x / I\_x$

$\dot{\omega}y = \ddot{\theta}y = \tau\_y / I\_y$

$\dot{z}\_b = vb$

$\dot{v}\_b = \ddot{z}\_b = a\_base(t) + (-F\_z + F\_ground) / m\_base$

These eight first-order ordinary differential equations are assembled in the function f(t, y) and integrated in time using an adaptive RKF45 scheme.

7. Base Excitation Waveform

The base acceleration input is specified as a simple pulse with user-defined peak amplitude A (in G) and pulse duration T. The code supports four waveform shapes:

- Half-sine
- Triangular
- Sawtooth
- Rectangular (step pulse)

For a given time t in the interval [0, T], the base acceleration in G is evaluated as:

- Half-sine: $a\_base,G(t) = A \sin(\pi t / T)$

- Triangular: rises linearly from 0 to A at T/2, then falls linearly back to 0 at T
- Sawtooth: $a\_base,G(t) = A (t / T)$
- Rectangular: $a\_base,G(t) = A$ for $0 \le t < T$, and 0 outside the pulse

For $t < 0$ or $t > T$ the base acceleration is taken as zero. The selected waveform is also rendered in a small panel on the 3D view for visualization, along with the approximate vertical, roll, and pitch natural frequencies derived from the stiffness and mass properties.

8. Time Integration: RKF45

The equations of motion are integrated using a Runge–Kutta–Fehlberg (RKF45) scheme with adaptive time stepping. The implementation uses the classical Cash–Karp coefficients to compute both a 4th-order and a 5th-order estimate over each step:

- • y^4 — 4th-order solution
- • y^5 — 5th-order solution

The difference between y^4 and y^5 is used to estimate the local truncation error. A combined relative/absolute error norm is computed across all state components:

$$err = \sqrt{ (1/n) \Sigma [(y^5{}_i - y^4{}_i) / (atol + rtol \cdot \max(|y_i|, |y^5{}_i|))]^2 }$$

If err ≤ 1, the step is accepted and the solution is advanced. If err > 1, the step is rejected and repeated with a smaller time step. The code adjusts the step size h according to a standard control law:

h_new = h · 0.9 · (1 / err)^{0.2}
clamped to lie within specified minimum and maximum factors.

The initial time step is set to a small value (on the order of 10^{-4} s), and a minimum step size is enforced to avoid infinite rejection stalls. The simulation runs from t = 0 up to a final time t_end chosen as a function of the lowest natural period, limited to a maximum of about 20 seconds.

9. Post-Processing: Accelerations and Spring Reactions

During post-processing, the code recomputes the kinematics and spring forces at each stored time step using the helper function responseAtSample. This function returns:

• az — vertical acceleration of the top COM
• aCorners — vertical acceleration at each corner of the top
• Fi — vertical force in each corner spring (acting on the top)

Center acceleration is converted to units of G for plotting and max-value reporting:

accel_center,G = az / g

The corner accelerations are similarly converted to G, and the module tracks the maximum absolute value over the full response history for each point. These maxima are displayed in a small summary panel in the main UI:

• Center peak |accel| (G)
• Peak |accel| at each corner: (0,0), (Lx,0), (0,Ly), (Lx,Ly)

The time histories of center acceleration and corner spring reactions are also stored and used to draw two plots:

1) Top center acceleration vs time (in G)
2) Corner spring reaction forces vs time (in lb or N)

10. Interactive Plot Cursor and Exports

Both plots support an interactive time cursor. When the user moves or drags the mouse over a plot, the code converts the horizontal cursor position to a corresponding time value, interpolates the response histories, and displays the instantaneous values in a readout beneath the plots (time, center acceleration, and corner spring forces).

Two export options are provided:
• Export JSON — writes a JSON file containing the time history of center acceleration and

corner spring reactions for each step.
• Export CSV — writes a CSV file with columns for time, center acceleration (G), and corner spring reactions, along with the force units label (lb or N).

11. Assumptions and Limitations

The Base Shake model is intended as a simplified representation of a base-excited system and is subject to the following assumptions and limitations:

• The top body is rigid; no local deformation is modeled.
• Only one vertical translation and two small rotations are included.
• Springs and dampers are linear; stiffness and damping do not vary with displacement or velocity.
• Damping is modeled as classical viscous damping derived from a user-specified fraction of critical damping.
• Base excitation is limited to a single finite-duration pulse in the vertical direction.
• The base mass is treated as a lumped mass with a simple spring–damper support to ground.

Within these assumptions, the module provides a convenient way to examine how mass distribution, spring stiffness, damping, and base shock characteristics influence the vertical and rocking response of a supported rigid body.