Beam Optimizer

1. Overview

The Beam Optimizer is an interactive, browser-based finite element tool for static analysis and section sizing of Euler–Bernoulli beams. It supports multi-segment beams with variable material and geometric properties, nodal boundary conditions, distributed and concentrated loads, internal hinges, elastic springs, and discrete section optimization against allowable bending stress limits.

2. User Interface

The interface is organized around a central beam canvas with associated control rows and result plots. User interaction is primarily graphical, with property editing performed via modal dialogs.

2.1 Global Controls

- Segments: Number of beam segments (1–10).
- Total Length: Total beam length, distributed evenly on build but change be changed by the user.
- Build Beam: Initializes segments and nodes.
- Solve: Performs static finite element analysis.
- Sections Catalog: Opens the section database editor. Discrete sections can be given or a continuous range between two limits.
- Optimize Sections: Runs the section optimization routine.
- Export CSV: Exports numerical results.
- EI Height Scale: Visual scaling factor for EI-based segment drawing.

2.2 Beam Canvas Interaction

- Clicking a segment opens the Segment Properties editor.
- Clicking a node opens the Node Properties editor.
- Segment height is proportional to EI.
- Load arrows, supports, hinges, springs, and moments are rendered symbolically.

2.3 Segment Properties

Each segment supports specification of:
- Elastic modulus (E)
- Area moment of inertia (I)
- Cross-sectional area (A)
- Length (L)
- Section moduli (St, Sb)
- Linearly varying distributed loads (qL, qR)
- Material weight density (wd)

An integrated Section Calculator computes A, I, St, and Sb for common shapes (rectangular, circular, tube, I-beam, channel, T-beam).

2.4 Node Properties

Node inputs include:
- Boundary condition: free, pinned, or fixed
- Concentrated force (F)
- Concentrated moment (M)
- Translational spring (Kv)
- Rotational spring (Km)
- Internal hinge (moment release)
- Prescribed displacement (w_0) and/or rotation (θ_0)

2.5 Output Plots

After solution, the following plots are generated:
- Deflection vs. length
- Bending moment diagram
- Shear force diagram
- Bending stress (top and bottom fibers)

3. Solution Methodology

3.1 Structural Model

The solver is based on the Euler–Bernoulli beam theory with two degrees of freedom per node: transverse displacement and rotation. Internal hinges are handled by splitting rotational degrees of freedom while preserving shear transfer.

3.2 Finite Element Formulation

Each segment is modeled using a standard two-node beam element with a 4×4 stiffness matrix derived from EI/L^3 scaling. Distributed loads are incorporated using consistent nodal load vectors computed via Gaussian quadrature.

3.3 Assembly and Constraints

Global stiffness and load vectors are assembled from element contributions. Boundary conditions and prescribed displacements are enforced using direct elimination of constrained degrees of freedom.

3.4 Solution

The reduced linear system is solved using dense linear algebra routines (numeric.js). Nodal displacements and rotations are recovered, followed by internal element force calculation.

3.5 Postprocessing

Bending moments and shear forces are reconstructed along each element using equilibrium relations. Bending stresses are computed as $\sigma = M / S$, using top and bottom section moduli.

4. Discrete Section Optimization

The optimizer assigns each segment a cross-section selected from a discrete catalog. The catalog may be entered manually or generated via linear interpolation between two endpoint sections.

Optimization proceeds iteratively:
1. Solve the beam with current section assignments.
2. Measure maximum absolute bending stress per segment.
3. Increase section size if $|\sigma|$ exceeds $\sigma\_allow\cdot(1+tolOver)$.
4. Decrease section size if $|\sigma|$ is below $\sigma\_allow\cdot(1-tolUnder)$.
5. Repeat until convergence or iteration limit.

The procedure is heuristic and monotonic; it does not perform gradient-based or continuous optimization.

5. Assumptions and Limitations

- Linear elastic behavior.
- Small deflection Euler–Bernoulli theory (shear deformation neglected).
- Static loading only (no inertia or dynamics).
- Stress checks limited to bending stress.
- Discrete optimization may converge to local optima.

6. References

1. Gere, J. M., and Timoshenko, S. P., Mechanics of Materials.
2. Cook, R. D. et al., Concepts and Applications of Finite Element Analysis.
3. Bathe, K. J., Finite Element Procedures.
4. 5. Zienkiewicz, O. C., and Taylor, R. L., The Finite Element Method.