Method Overview — Nonlinear Neo-Hookean, Axisymmetric Finite Elements

Problem

Bonded compression of an axisymmetric rubber part (cylinder / cone / curved cone). Bottom face bonded and fixed; top face bonded and driven downward by a prescribed displacement. The analysis computes deformed shape, shear distribution, reaction load, and effective stiffness.

Material model (compressible neo-Hookean)

Shear modulus: $\mu = G$
Bulk modulus: $\kappa = K\_bulk$ ($\kappa \gg \mu$ to suppress volumetric strain)

Strain energy: $W = (\mu/2)\cdot(I_1 - 3 - 2\cdot\ln J) + (\kappa/2)\cdot(\ln J)^2$
Nominal stress (PK1): $P = \mu\cdot(F - F^{-T}) + \kappa\cdot\ln(J)\cdot F^{-T}$

Kinematics & axisymmetry

• Mesh is built in the meridional (r–z) plane using 4-node bilinear quads; DOFs per node: (u_r, u_z).
• Deformation gradient F is evaluated at Gauss points; circumferential stretch $\lambda\_\theta = r/R$ enters $J = \det(F)\cdot\lambda\_\theta$.
• All element integrals include the axisymmetric weight $2\pi\cdot r$.

Weak form & discretization

From virtual work, assemble the internal residual:
$R(u) = \int B^T \cdot P(F(u)) \cdot (2\pi r)\, dV$
where B maps nodal DOFs to gradients in (R,Z,θ). A stabilized secant-type tangent
$K\_t \approx \int B^T\cdot D\_sec\cdot B \cdot (2\pi r)\, dV$
is used to accelerate convergence while keeping the formulation robust for browser execution.
The diagonal entries of D_sec scale with (μ, κ).

Boundary conditions

• Bottom (z=0): bonded → u_r=0, u_z=0.
• Top (z=H): bonded, imposed displacement → u_r=0, u_z=−Δ.
• Axis (r=0): handled naturally by axisymmetric formulation.

Solution strategy

• Load stepping: increase prescribed compression to the target percentage of height.
• Newton iterations: solve $K\_t\cdot\Delta u = -R$ until residual norm is small.
• Linear solver: preconditioned conjugate gradient (diagonal preconditioner).

Outputs

- Reaction force on the top plate; effective stiffness $k = F/\delta$.
- Deformed outline and a color map of shear γ_{rz}.
- Load–deflection curve from converged steps.

Notes on near-incompressibility

A large κ (bulk modulus) enforces small volumetric strain ($J \approx 1$). This prevents volume locking while allowing realistic bulging under bonded compression.

References

Bonet, J., & Wood, R. D. Nonlinear Continuum Mechanics for Finite Element Analysis, 2nd ed.
Bower, A. F. Applied Mechanics of Solids.
Ogden, R. W. Non-Linear Elastic Deformations.