Methods — Cooling Shrinkage & Shear Strain (Axisymmetric Thermo-Elastic FEM)

Purpose.

Predict bonded rubber disk shrinkage on cool-down from mold temperature Tm to room temperature Tr, and visualize the resulting shear strain field γrz in r–z.

1) Geometry & Mesh

- Axisymmetric body with options: cylinder, straight cone, or "curved cone" (two-k bump on inner/outer radii).
- Meridional mesh: structured Q4 bilinear elements (nr×nz), nodes linearly distributed between inner/outer radii at each z.
- Axisymmetric mapping uses the current local radius R in quadrature weights (2πR·detJ).

2) Material & Thermal Strain

- Small-strain, isotropic linear elasticity with near-incompressibility: given shear modulus G and ν=0.49 ⇒ Lamé parameters λ = (2Gν)/(1−2ν), μ = G.
- Uniform temperature drop ΔT = Tm − Tr.
- Isotropic thermal strain vector (axisym ordering [rr, zz, θθ, rz]) is εth = [αΔT, αΔT, αΔT, 0].

3) Element Formulation

- Q4 axisymmetric with 2×2 Gauss integration. Shape-function gradients are mapped via the Jacobian.
- Strain–displacement matrix B includes the hoop term εθθ = Σ(Ni·ur)/R.
- Constitutive matrix D (axisym, linear isotropic). Element stiffness: Ke = ∫(BTDB)·(2πR detJ) dξ dη.
- Thermal load vector via initial-strain route: fth,e = −∫(BT D εth)·(2πR detJ) dξ dη.

4) Boundary Conditions

- Bonded ends: All DOF fixed on z=0 and z=H (ur=uz=0); this suppresses axial free-shrink and induces shear.
- Free ends: Only one anchor to remove rigid motion (uz=0 at a corner node); the rest are free.

5) Global Solve

- Dense assembled K and f (including thermal). PCG with diagonal preconditioning solves K u = f.
- Nodal displacements split as ur, uz.

6) Shrinkage & Shear Outputs

- Dimensional shrinkage is read directly from boundary nodes: inner/outer radii after cooling $r_{inner}(z) = R_{inner} + u_r$, $r_{outer}(z) = R_{outer} + u_r$; axial contour $z_{after}(z) = z + u_z$. The plotted "AFTER" outline uses a visual scale factor for clarity.
- Shear strain γ_{rz} is computed at each element center from the displacement field: $\gamma_{rz} = \partial u_r/\partial z + \partial u_z/\partial r$, using B-matrix gradients (absolute value for coloring). Contours are mirrored about r=0 for the full cross-section view.
- A quick analytic status estimate (not used for the final plot) assumes uniform free shrink: $s_r=\alpha\Delta T$, $s_z=\alpha\Delta T$ (free ends) or 0 (bonded ends), and reports $|s_r-s_z|$ as a crude shear proxy.

7) Units

- Two systems: US (lb, in, °F) or SI (N, mm, °C). Lengths/G/temps are entered in the chosen system.
- CTE α is per-degree in the chosen temperature unit. When switching units, α is converted by 9/5 or 5/9 so $\alpha\Delta T$ stays consistent.

Assumptions & Notes

- Small strains; homogeneous, linear elastic solid with $\nu=0.49$ (near-incompressible). No cure kinetics or viscoelasticity.
- Uniform ΔT; no radial/axial thermal gradients; no contact/friction modeling at walls; axisymmetric only.
- Q4 elements with $\nu\approx0.5$ can exhibit volumetric locking; selective/reduced integration or mixed u–p would further improve fidelity.