State-of-Cure Method Used

Thermal Field (Axisymmetric FEM)

• Four-node bilinear axisymmetric elements with 2×2 Gauss integration.
• Assembles the transient conduction system

$$\mathbf{C \cdot \dot{T} + K \cdot T = Q}$$

where **C** is the heat-capacity matrix ($\rho\ c_p\ N^T N$), **K** is the conductivity matrix ($k\ \nabla N \cdot \nabla N$), and **Q** is the internal heat generation from cure.
• **Time stepping:** backward-Euler implicit:

$$(C/\Delta t + K)\ T^{n+1} = (C/\Delta t)\ T^n + Q^{n+1}.$$

• **Boundary conditions:** Dirichlet (fixed temperature) at all outer nodes or OD-only at $T_m$old, as selected.
• Axisymmetric weighting includes the 2 π r term in each element integral.

Cure Kinetics (Autocatalytic / Kamal–Sourour Form)

Degree of cure $\alpha \in [0, 1]$ evolves as

$$d\alpha/dt = (\mathbf{k_1 + k_2\ \alpha^p})\ (1 - \alpha)^p$$

with Arrhenius temperature dependence

$$\mathbf{k_i = A_i\ \exp(-E_i\ /\ R\ T)} \quad \text{for } i = 1, 2.$$

The rate is computed at every node from the current temperature field.

Thermo-Chemical Coupling (Exotherm)

Internal heat generation per unit volume:

$$\mathbf{\dot{q} = \rho\ H_r\ (d\alpha/dt)}$$

This term enters the right-hand side Q of the conduction equation each time step.

Stability and Step Control

A per-step limit $\mathbf{\Delta\alpha_{max}}$ ensures no node advances more than this fraction per time step, keeping the coupled system stable under strong exotherm.

Outputs and Tracking

• Snapshots of temperature (T) and degree of cure (α) are stored at 0%, 10%, 20%, … 90% minimum cure and at completion.
• Each step reports:
  – Minimum α
  – % of nodes with α ≥ 0.90
  – Maximum temperature
  – A color-mapped r–z plot (blue → red) of α.

Why This Approach Works Well

• The Kamal–Sourour model is standard for thermosets and rubbers: compact, fits DSC data, and captures both autocatalysis and diffusion-limited slowdown.
• Implicit heat stepping with Dirichlet BCs is unconditionally stable.
• The $\Delta\alpha_{max}$ limit provides simple adaptive damping against runaway.

Key References

1. **Kamal, M. R. & Sourour, S. (1973)**. *Kinetics and thermal characterization of thermoset cure.* Polymer Engineering & Science.
2. **Bogetti, T. A. & Gillespie, J. W. (1992)**. *Two-dimensional cure simulation of thick thermosetting composites.* Journal of Composite Materials.

3. **Advani, S. G. & Sozer, E. M. (eds.) (2010/2011)**. *Process Modeling in Composites Manufacturing.* CRC Press.
4. **Incropera et al.** *Fundamentals of Heat and Mass Transfer.* Wiley.