

Bolted Joint Bolt Sizing Module

This document provides a complete theoretical, mathematical, and computational foundation for the Bolted Joint Preload Sizing Module. All governing equations are presented in derivation form consistent with classical mechanical design texts.

1. Introduction

This manual documents the complete analytical basis used to determine the minimum bolt size required to prevent joint separation while ensuring proof strength compliance under torque scatter and friction variability.

The formulation follows classical bolted joint theory as presented in:

- Shigley & Mischke – Mechanical Engineering Design (5th Edition)
- Bickford – Design and Behavior of Bolted Joints
- NASA RP-1228 Fastener Design Manual

2. Elastic System Model

The joint is modeled as two springs in series:

Bolt stiffness: k_b

Member stiffness: k_m

Load fraction carried by the bolt under external tensile load P :

$$C = k_b / (k_b + k_m)$$

Separation prevention requirement:

$$F_{i,min} \geq (1 - C) P$$

Maximum bolt load under service condition:

$$F_{b,max} = F_{i,max} + C P$$

Proof constraint:

$$F_{b,max} \leq \phi_{proof} F_{proof}$$

3. Bolt Stiffness Derivation

Axial deformation relation:

$$\delta = F L / (A E)$$

Bolt consists of two elastic segments:

- Unthreaded shank (area A_{shank})
- Threaded portion (effective area ηA_t)

Shank area:

$$A_{shank} = \pi d^2 / 4$$

Thread tensile stress area:

A_t = tensile stress area per standard tables

Thread compliance factor:

$\eta = 0.80\text{--}1.00$ (default 0.90)

Total bolt compliance:

$C_b = L_{\text{shank}}/(A_{\text{shank}} E) + L_{\text{thread}}/(\eta A_t E)$

Bolt stiffness:

$k_b = 1 / C_b$

4. Member Stiffness – Shigley 30° Pressure Cone

Following Shigley, compression beneath the head and nut is modeled as a truncated pressure cone with half-angle $\alpha = 30^\circ$.

Diameter variation along depth x :

$D(x) = D_b + 2 x \tan(\alpha)$

Cross-sectional area with clearance hole d_h :

$A(x) = (\pi/4)[D(x)^2 - d_h^2]$

Elemental compliance:

$d\delta = dx / (E A(x))$

Integrated compliance per side:

$C_{\text{half}} = \int dx / (E A(x))$

Total member stiffness:

$k_m = 1 / (C_{\text{top}} + C_{\text{bottom}})$

Gasket or O-ring layers are modeled as direct linear springs in series.

5. Torque–Preload Relationship

Total tightening torque consists of:

- Thread torque component
- Bearing friction component

Thread helix angle:

$\lambda = \arctan(p / (\pi d_2))$

Thread friction angle:

$$\varphi = \arctan(\mu_{\text{thread}} \sec 30^\circ)$$

Total torque expression:

$$T = F \left[\left(\frac{d_2}{2} \right) \tan(\lambda + \varphi) + \mu_b \left(\frac{D_b}{2} \right) \right]$$

Preload inversion:

$$F = T / g(\mu_{\text{thread}}, \mu_{\text{bearing}})$$

6. Torque Band Methodology

Lower preload bound (separation check):

- Maximum μ_{thread}
- Maximum μ_{bearing}
- Minimum torque ($T - \text{tolerance}$)

Upper preload bound (proof check):

- Minimum μ_{thread}
- Minimum μ_{bearing}
- Maximum torque ($T + \text{tolerance}$)

Feasible torque band condition:

$$T_{\text{required}} \leq T_{\text{allowable}}$$

If no overlap exists, the bolt size is rejected.

7. Clearance Hole Standards

Metric: ISO 273 – Close, Normal, Loose fits.

Inch: ASME/ANSI B18 practice.

Hole diameter directly affects pressure cone geometry.

8. Minimum Size Scan Algorithm

1. Iterate bolt sizes in ascending diameter.
2. Compute k_b and k_m .
3. Determine torque band feasibility.
4. First feasible size is selected.
5. Torque is set to midpoint of feasible band.

9. Validation & Benchmarking

Validation performed against textbook problems and independent calculation tools.

10. Assumptions and Limitations

- Linear elastic behavior
- Static tensile loading
- No embedment relaxation
- No prying or bending effects
- No thermal effects
- Uniform friction coefficients

11. References

1. Shigley, J.E., Mischke, C.R., Mechanical Engineering Design, 5th Ed.
2. Bickford, J.H., Design and Behavior of Bolted Joints.
3. NASA RP-1228 Fastener Design Manual.
4. ASME B18.2.1 and B18.2.2.
5. ISO 273 – Clearance Holes.