Channeled Plate Heat Sink Module — Description and Methods

1. Overview and Intended Use

This module estimates the steady-state temperature field and pressure drop for a flat plate heat sink cooled by parallel rectangular channels machined into the plate. The tool is intended for early-stage design of liquid-cooled cold plates and channelled heat sinks where the channel layout is uniform and the dominant thermal resistance is conduction in the plate plus convection to the coolant.

The user defines the plate planform, material, coolant type, flow rate, and inlet temperature. Up to five rectangular heat sources can be placed on the plate using a simple drawing interface. The module then sweeps over a range of possible channel counts, computes the peak plate temperature and coolant-side pressure drop for each case, and presents the results in a compact chart and status table. For any selected channel count, the user can render a temperature contour map over the plate surface.

1.1 User Interface Structure

The HTML layout is divided into two primary columns:

Left column — setup and solution controls:

- Setup — Plate & Coolant: plate width and length (mm), plate material (Aluminum, Copper, Stainless Steel, FRP), coolant type (Water or 50/50 glycol mixture), volumetric flow rate (L/min), and coolant inlet temperature (°C).
- Solution — Layout & Heat: channel depth b, minimum wall thickness t, computed channel width w, plate thickness, laminar Nusselt-number boundary condition, estimates of the feasible minimum/maximum number of channels, a heat source list editor, solve and contour buttons, and CSV/JSON export buttons.

Right column panels include:

- Plate Grid: a scaled plan-view grid of the plate used to place heat sources by click-and-drag. Heat sources are shown as colored rectangles with labels and power values.
- Channel End View (Min #): a schematic cross-section of the plate showing the minimum feasible number of channels, wall thickness, channel width, depth, and plate thickness.
- Solve Status: a table reporting, for each channel count N in the sweep, the boundary-condition tag (BC), Reynolds number Re, Nusselt number Nu, convection coefficient h, pressure drop ΔP, and maximum plate temperature Tmax.
- Temperature Contours: a canvas where the computed plate temperature field is rendered as a color contour plot for a selected channel count.

2. Physical Model and Governing Equations

The model treats the plate as a 2D conduction domain in the plane of the plate (x–y), with uniform properties and constant thickness. Heat sources are applied as distributed surface heat fluxes over user-defined rectangular regions. Convective cooling is applied wherever a channel is present, and the coolant temperature under each channel is obtained from a 1D energy balance along the flow direction.

2.1 Geometry and Discretization

The plate planform is rectangular with width W (mm) and length L (mm). Internally, the solver converts these to SI units and discretizes the plate into an $nx \times ny$ structured grid, where nx is fixed at 120 and ny is chosen to preserve aspect ratio ($ny \approx nx \cdot W/L$, with a minimum of 12 cells). Each cell has area $A_{cell} = \Delta x \cdot \Delta y$.

Up to five rectangular heat sources can be specified. Each heat source i has a total power Wi and occupies a rectangular region in the plate coordinates. The module builds a heat generation map $q''(x,y)$ (W/m^2) by distributing Wi uniformly over its area and summing contributions from all heat sources. At the grid level, each cell receives a uniform volumetric term qpp[j][i] (W/m^2) based on whether the cell center falls inside the heat source rectangle.

2.2 Channel Geometry and Hydraulic Parameters

Channels are rectangular, with depth b (mm), width w (mm), and separated by walls of thickness t (mm). For a given plate width W and trial channel count N, the channel width is computed assuming fixed wall thickness:

$$w = (W - (N + 1)\, t) / N$$

If this expression yields a non-positive width, the channel count is reduced until a feasible layout is obtained. The end-view panel uses the minimum-feasible channel count to illustrate the geometry.

For thermal/hydraulic calculations, the channel cross-sectional area A and hydraulic diameter D_h are:

$$A = w \cdot b$$
$$D_h = 2A / (w + b)$$

The total volumetric flow rate Q_total (m^3/s) is obtained from the user-specified flow in L/min. For a given N, the per channel flow is Qch = Qtotal / N, and the mean velocity is v = Qch / A.

The Reynolds and Prandtl numbers are defined as:

$$Re = (\rho\, v\, Dh) / \mu$$
$$Pr = (\mu\, cp) / k$$

where ρ is density, μ is dynamic viscosity, k is thermal conductivity, and c_p is specific heat of the coolant. The module provides built-in constant-property values for water and a 50/50 glycol mixture at representative temperatures.

2.3 Heat Transfer Correlations

For laminar flow ($Re < 2300$), the module uses correlations for fully-developed flow in rectangular ducts with various thermal boundary conditions:

- H2-4: all walls at uniform heat flux (isoflux).
- T: all walls isothermal; implemented as a conservative multiplier on the H2-4 Nusselt value.
- H2-1: top wall isoflux, other walls adiabatic.

The base H2-4 correlation for a rectangular duct uses a shape factor derived from the aspect ratio $\alpha = \min(w,b)/\max(w,b)$:

$$\beta = (1 - \alpha) / (1 + \alpha)$$
$$b\_r = 1 - 2.0421\,\beta + 3.0853\,\beta^2 - 2.4765\,\beta^3 + 1.0578\,\beta^4 - 0.1861\,\beta^5$$
$$NuH2\text{-}4 = 8.235 \cdot br$$

The other laminar Nusselt numbers are obtained via simple multipliers:

$$NuT = 0.915 \cdot NuH2\text{-}4 \quad \text{(all walls isothermal)}$$
$$NuH2\text{-}1 = 1.18 \cdot Nu\_H2\text{-}4 \quad \text{(top wall isoflux, others adiabatic)}$$

For turbulent flow ($Re \geq 2300$), the module uses a Dittus–Boelter type relationship for fully-developed internal flow:

$$Nuturb = 0.023 \cdot Re\text{^}0.8 \cdot Pr\text{^}0.4$$

In all cases, the convective heat transfer coefficient is computed as $h = Nu \cdot k / Dh$.

2.4 Pressure Drop Model

The pressure drop across the heated length L is estimated using a friction factor correlation and the single-phase momentum equation. The module uses:

- • Laminar flow ($Re < 2300$): $f = 64 / Re$
  • Transitional/turbulent: an implicit smooth tube approximation of the Colebrook type relation, written in closed form.

Once f is known, the channel pressure drop is:

$$\Delta P = f \cdot (L / Dh) \cdot (\rho\, v^2 / 2)$$

The status table and chart report ΔP in kPa.

2.5 Plate Energy Equation and Discretization

The plate is modeled as a uniform, constant thickness slab of thickness tplate and thermal conductivity ks. The in-plane temperature T(x,y) satisfies a steady 2D conduction equation with volumetric heating and convection to the coolant wherever a channel is present:

$$-\nabla\cdot(\text{ks tplate } \nabla T) + \text{h Acellmask} \cdot (T - Tb) = q'' \text{ Acell}$$

Here Acellmask is the projected area of a cell that lies under a channel, and Tb is the local bulk coolant temperature in the channel segment above that cell. Outside the channels, the convective term is zero. The code implements a finitevolume style five-point stencil on the structured grid, with east/west and north/south conductances proportional to ks tplate/Δx and ks tplate/Δy respectively.

The convective terms are accumulated cell-by-cell using precomputed masks for each channel. For each cell, the coefficient matrix diagonal is augmented by h Acellmask and the right-hand side receives h Acellmask Tb. The result is a large sparse linear system A T = b, where T is the vector of cell temperatures.

2.6 Linear Solver

To avoid explicitly forming the global matrix, the module uses a matrix-free conjugate-gradient (CG) solver. The applyA(x, y) routine computes y = A x by looping over grid cells, applying the five-point conduction stencil and the local convective diagonal term. A standard CG iteration with residual-based convergence is used, with user-independent defaults for maximum iterations and tolerance. The initial guess is a uniform field at the coolant inlet temperature.

2.7 Coolant Energy Balance and Coupling

For each channel, the coolant bulk temperature Tb(x) is updated from a 1D energy balance that couples the local convective heat flux from the plate to the coolant enthalpy rise. Along the channel:

$$\dot{m}\ cp\ dTb/dx = \sum\text{cells } h\ (Tplate - Tb)\ Acell$$

Here ṁ is the mass flow rate per channel, cp is specific heat, and the summation runs over grid cells that lie under that channel station. The code discretizes this equation with a marching scheme in x, using the current plate temperature field T to evaluate the heat flux and updating Tb station by station. A relaxation factor is applied to stabilize convergence.

The overall plate coolant coupling is handled iteratively:

1. Initialize Tb for all channels at the coolant inlet temperature.
2. Build the plate operator A and right-hand side b using the current Tb.
3. Solve A T = b using CG to obtain an updated plate field.
4. March the coolant energy balance along each channel using the new plate temperatures to update Tb.

5. Repeat steps 2–4 a small number of times (two iterations in the current implementation).

This procedure yields an approximate conjugate heat transfer solution for the coupled plate and coolant system at steady state.

2.8 Channel Sweep and Post-Processing

For each run, the module constructs a set of candidate channel counts N between a minimum and maximum value based on simple geometric rules (approximately W/8 and W/3, in mm). For each N in this set it performs the hydraulic, thermal, and conjugate plate–coolant calculations described above. It extracts, for each case:

- Reynolds number Re
- Nusselt number Nu
- Convection coefficient h
- Pressure drop ΔP (kPa)
- Peak plate temperature Tmax (°C)

These values are stored and displayed in the status table. The chart pane plots Tmax versus N (blue curve) and ΔP versus N (red curve) on dual y-axes. The user can click on a blue data point to select a particular N for contour plotting.

2.9 Temperature Contour Rendering

For a selected channel count, the stored temperature field is interpolated onto the contour canvas. The code uses bilinear interpolation from the nx × ny grid to the pixel grid of the canvas, and maps the non-dimensionalized temperature (from Tmin to Tmax) to a simple "jet-like" blue–cyan–yellow–red colormap. Axes, tick marks, and labels are drawn to match the physical plate dimensions in millimetres. The contour summary displays the channel count, boundary-condition tag, and the range Tmin–Tmax in °C.

3. How to Use the Module

A typical workflow is:

1. 1. Define the plate and coolant:

  - Enter the plate width and length in millimetres.
  - Choose the plate material (the thermal conductivity is set internally).
  - Select the coolant (water or 50/50 glycol mixture).
  - Enter the total coolant flow rate in L/min and the inlet temperature in °C.

2. 2. Define channel layout parameters:

  - Enter the channel depth b and minimum wall thickness t.
  - Specify the plate thickness.
  - Choose the laminar boundary condition to use when Re < 2300. The all-walls-isothermal option is conservative and generally produces the highest predicted temperatures.

3. 3. Add heat sources:

- Click the "Add" button in the Heat Sources panel.
- On the Plate Grid, drag out a rectangle representing the footprint of the heat source.
- When prompted, enter the power for that region in watts.
- Repeat up to a maximum of five heat sources. The list view shows each source index, power, and footprint size; individual sources can be deleted as needed.

4. 4. Solve and review the sweep:

- Click "Solve". The solver sweeps over a small set of feasible channel counts, solves the conjugate problem for each case, and fills the status table.
- The chart shows peak plate temperature versus N (blue) and pressure drop versus N (red). Use this to assess the trade-off between thermal performance and pumping requirement.

5. 5. Inspect temperature contours for a selected N:

- Click the "T Contours" button to enter selection mode.
- On the chart, click on one of the blue Max-T data points. The corresponding plate temperature field is rendered in the Temperature Contours panel.
- Review the spatial temperature distribution, hot-spot locations, and overall temperature range.

6. 6. Export results:

- Use the "CSV" button to export the sweep summary (N, BC, Re, Nu, h, ΔP, Tmax) for further post-processing.
- Use the "JSON" button to export both the inputs and the sweep results in a machine-readable format, including minimum temperature where available.

4. Assumptions and Limitations

The module is designed as an engineering-level estimator rather than a detailed CFD or finite-element model. Key assumptions and limitations include:

- Steady-state behaviour only; no transient effects are modeled.
- Single-phase liquid flow with no boiling, cavitation, or phase change.
- Constant fluid properties for each coolant type, independent of local temperature.
- Constant plate thermal conductivity and uniform plate thickness.
- The plate is modeled with in-plane conduction only; through-thickness temperature gradients are lumped into an effective conductance $k_s \cdot t_{plate}$.
- The flow is assumed fully developed in both velocity and temperature senses; entrance effects are not modeled.
- The laminar Nusselt correlations are strictly valid over a limited range of aspect ratios and Re; outside those ranges, results should be treated as approximate.
- The turbulent Dittus–Boelter correlation is intended for smooth tubes and may be less accurate for very small or highly rectangular channels.

- Flow distribution is assumed uniform among channels; manifold and header losses are not modeled, and maldistribution effects are ignored.
- Radiation and heat loss to the environment are neglected; all heat is assumed to flow into the coolant.
- Heat sources are assumed to apply a uniform heat flux over their footprints; any finer-scale non-uniformity is not resolved.
- The coupling between plate and coolant is treated with a small fixed number of iterations, providing an approximate conjugate solution rather than fully converged nonlinear coupling.

4.1 Recommended Use Ranges

Within these limitations, the module is most appropriate for preliminary design studies where the following conditions are broadly met:

- Plate sizes from a few centimetres up to a few hundred millimetres.
- Total heat loads from tens to several hundred watts.
- Coolant flow rates on the order of 0.5–5 L/min of water or similar liquids.
- Channel aspect ratios (b/w) near unity or moderately elongated; very extreme aspect ratios may reduce correlation accuracy.

5. References and Further Reading

The following references provide background for the correlations and methods used in this module:

- • Incropera, F. P., DeWitt, D. P., Bergman, T. L., & Lavine, A. S., "Fundamentals of Heat and Mass Transfer," 6th or later edition, Wiley.
- • Shah, R. K., & London, A. L., "Laminar Flow Forced Convection in Ducts," Advances in Heat Transfer, Supplement 1, Academic Press.
- • Kays, W. M., & Crawford, M. E., "Convective Heat and Mass Transfer," McGraw-Hill.
- • White, F. M., "Fluid Mechanics," McGraw-Hill, for friction-factor and internal-flow fundamentals.