Plenum / Fan –Simple Flow + Branch Split + Velocity Blocks

This document summarizes the use, solution method, and key references for the "Plenum / Fan – V2 Simple Flow + Branch Split + Velocity Blocks" module. The tool matches a user-specified fan curve to a plenum with a finned heat sink, estimates the flow distribution among parallel flow paths, and visualizes both the fan–system operating point and velocity distribution over the heat sink.

1. Purpose and Scope

The module is intended for preliminary design and comparison studies of plenum-fed heat sinks and manifolds. It operates in IP display units (in, inH_2O, cfm) while performing all internal calculations in SI units. The primary goals are to:

- Determine the operating point (Q^*, Δp^*) where the fan curve intersects the system curve.
- Estimate inlet and channel velocities, Reynolds numbers, and fan air power.
- Model how flow splits between three parallel regions: channels through the heat sink, a top gap above the fins, and side gaps around the heat sink.
- Provide a 1D velocity profile Vch(x) along the fin length for the channel region and visualize relative velocities in a simple 3D view.

2. Units and Conventions

User inputs and displayed results use IP units, while the solver converts to SI internally:

- Flow rate: Q in cfm (IP) → Q in m^3/s via CFM_TO_M3S = 0.00047194745.
- Length: inches → meters via IN_TO_M = 0.0254.
- Pressure: inH_2O → Pa via INH2O_TO_PA = 249.0889.
- Density ρ in kg/m^3 and viscosity μ in Pa·s are entered directly in SI.

The solver assumes steady, incompressible, single-phase air flow. All pressure drops are modeled using Darcy–Weisbach friction plus lumped minor loss coefficients.

3. User Inputs and Outputs

3.1 Air Properties

The Air panel sets the fluid properties used in Reynolds number and pressure-drop calculations:

- Density ρ (kg/m^3) – typically around 1.2 kg/m^3 for air at room conditions.
- Viscosity μ (Pa·s) – default 1.82×10^{-5} Pa·s (dynamic viscosity of air).

3.2 Plenum and Inlet Geometry

The Plenum & Inlet panel defines the rectangular plenum and the blower / fan inlet opening (in IP units):

- Plenum dimensions: width W, height H, length L (in).
- Inlet aperture: width w, height h, and axial position x from the plenum start (in).

These dimensions are used to compute cross-sectional areas, hydraulic diameters, and plenum velocity for Reynolds number and friction calculations. The same geometry is also used in the 3D view.

3.3 Heat Sink Geometry

The Heat Sink panel describes a finned heat sink placed within the plenum, modeled as a bank of parallel channels:

- Fin height b (in) – vertical projection of fins into the flow.
- Fin thickness t (in) and spacing s (in) – repeated pitch is s + t.
- Fin length Lf (in) – flow length through the channels.
- Heat sink lead edge xHS (in) – axial location where the fins start.
- Heat sink width WHS (in, ≤ W) – width occupied by the fin bank. This implicitly sets side-gap widths.

The model automatically computes the number of channels, total channel flow area, and hydraulic diameters for the heat sink region. It also identifies the top gap above the fins and the side gaps around the heat sink when WHS < W or b < H.

3.4 Fan Curve (IP)

The Fan Curve panel accepts an arbitrary static fan curve as a set of (Q, Δp) points in IP units:

- Q in cfm; Δp in inH_2O.
- Rows can be added or removed interactively.

The solver sorts the fan points by flow rate and uses piecewise-linear interpolation to obtain Δp_fan(Q) for any intermediate Q. Extrapolation is linear between the first two and last two points if Q lies outside the tabulated range.

3.5 Velocity Slices for Vch(x)

The Velocity Distribution panel includes the number of slices nSlices along the heat sink length. This controls the resolution of the 1D plenum-over-HS manifold model. Typical values range from 10–40 slices.

3.6 Key Outputs

When the Solve button is pressed, the module computes and displays:

- Operating flow rate Q* (cfm) and back pressure Δp* (inH_2O).

- Inlet velocity Vin (ft/s) based on fan aperture area.
- Average heat sink channel velocity Vch,avg (ft/s).
- Reynolds number in the plenum Repl and in the channels Rech.
- Fan air power (HP) based on Q* and Δp*.
- Vch(x) profile (m/s) along the fin length, plotted vs x / Lf.
- Fan and system curves overlaid on a Q–Δp graph including the operating point.
- 3D geometry view with velocity-colored blocks for channels, top gap, and side gaps, plus a legend in ft/s.

4. Core Solution Method

4.1 Fan Curve Interpolation

The fan curve is defined in IP units as discrete points (Q, Δp). The solver:

- Sorts the points by flow rate Q.
- Uses linear interpolation between adjacent points to obtain Δp_fan(Q).
- Uses linear extrapolation between the first two and last two points if Q lies below or above the tabulated range.

This provides a continuous fan characteristic Δp_fan(Q) suitable for locating the intersection with the system curve.

4.2 System Pressure Drop Model

The system pressure drop Δp_sys(Q) is modeled as the sum of three components, all evaluated in SI units:

- Inlet loss at the fan opening.
- Parallel branch network losses (channels, top gap, and side gaps).
- Exit loss at the plenum outlet.

The Darcy–Weisbach equation and minor loss coefficients are used to estimate the pressure drop in each region.

4.2.1 Inlet Loss

For a given volumetric flow Q (m^3/s), the velocity through the inlet aperture is:

$$Vin = Q / Ain \ , \ Ain = wIn \times hIn$$

The associated head loss is modeled as a simple minor loss:

$$\Delta p\_in = K\_inlet \cdot \tfrac{1}{2} \rho \, Vin^2$$

with K_inlet ≈ 1.0 as a representative inlet loss coefficient.

4.2.2 Three-Branch Plenum / Heat Sink Network

The plenum region over the heat sink is idealized as three parallel flow paths that share a common pressure drop:

- Channel branch (ch): flow through the fin channels, length Lf, area Ach, hydraulic diameter $D_{h,ch}$.
- Top branch (top): flow in the top gap above the fins, length L, area A_top, hydraulic diameter $D_{h,top}$.
- Side branches (side): combined flow in side gaps around the heat sink, length L, area A_side, hydraulic diameter $D_{h,side}$.

If the geometry degenerates (for example, no fins or no gaps), the model falls back to treating the entire plenum as a single duct.

For each branch i, a quadratic resistance law is established:

$$\Delta p_i \approx k_i Q_i^2$$

where Q_i is the branch flow rate and k_i is an effective flow resistance coefficient derived as follows:

- Assume a reference velocity Vref = 1 m/s in the branch.
- Compute Reynolds number $Re_{ref} = \rho V_{ref} D_h / \mu$.
- Compute friction factor f_ref using a composite correlation:
  $f = 64 / Re$ for $Re < 2300$ (laminar)
  $f = 0.3164 / Re^{0.25}$ for $Re \geq 2300$ (turbulent, Blasius).
- Compute total pressure drop at Vref as $\Delta p_{ref} = \frac{1}{2} \rho V_{ref}^2 [f_{ref} (L / D_h) + K_{minor}]$.
- Relate branch flow to velocity via $Q_{ref} = A V_{ref}$ and form $k_i = \Delta p_{ref} / Q_{ref}^2$.

The minor loss constant K_minor lumps entrance, exit, and bundle effects for each branch type. Representative values of order unity are used (e.g., K_CH_MINOR ≈ 2.0 for channels).

Because the three branches are in parallel, they experience a common pressure drop Δp_branch. For a given total system flow Q, the branch flows are allocated according to the relative resistance:

$$Q_i \propto 1 / \sqrt{k_i}, \quad \text{with} \quad \Sigma Q_i = Q$$

This is implemented by computing $1/\sqrt{k_i}$ for each branch, normalizing to the total, and assigning branch flows and velocities accordingly. The common Δp_branch is then obtained from any branch via $\Delta p_{branch} = k_i Q_i^2$.

4.2.3 Exit Loss

The plenum outlet is treated as an additional minor loss based on the full plenum cross-sectional area Aout = W × H:

$Vout = Q / Aout$

$\Delta p\_out = K\_exit \cdot \frac{1}{2} \rho Vout^2$

with K_exit ≈ 1.0 as a representative outlet loss coefficient.

4.2.4 Total System Curve

The system pressure drop as a function of Q (in SI units) is therefore:

$\Delta p\_sys(Q) = \Delta p\_in(Q) + \Delta p\_branch(Q) + \Delta p\_out(Q)$

This is converted back to inH_2O for plotting alongside the fan curve.

4.3 Operating Point Search

To find the operating point, the solver scans over flow rates from Q = 0 to the maximum tabulated fan flow. For each trial Q:

- Compute Δp_fan(Q) from the interpolated fan curve (inH_2O).
- Compute Δp_sys(Q) using the system model and convert to inH_2O.
- Evaluate the absolute difference |Δp_fan − Δp_sys| and keep the Q that minimizes this error.

The resulting Q* and Δp* define the fan–system operating point. These values are used for all downstream calculations (velocities, Reynolds numbers, and air power).

4.4 Reynolds Numbers and Fan Air Power

Reynolds numbers are computed using the standard definition $Re = \rho V Dh / \mu$, where V is the mean velocity and Dh is the hydraulic diameter of the relevant flow region. In particular:

- Re_pl is computed using the full plenum cross-section W × H and Dh based on that rectangle.
- Re_ch is computed using the channel velocity and hydraulic diameter for the fin passages.

The fan air power, neglecting fan efficiency, is computed as:

$P\_air = Q^* \cdot \Delta p^*$

where Q* is in m^3/s and Δp* in Pa. The result is converted to horsepower (HP) using 1 HP ≈ 745.7 W.

4.5 1D Velocity Profile Along the Heat Sink (V2 Model)

The velocity distribution Vch(x) over the heat sink is computed using a 1D plenum-over-HS manifold model along the fin length Lf. The heat sink region is divided into nSlices axial segments. The approach is:

- Define the plenum cross-section above the fins with area Apl (using the top-gap height $H - b$) and hydraulic diameter Dh_pl.
- Estimate an effective channel loss coefficient Kch,eff using the reference operating point and the computed channel friction plus end losses.
- Assume an unknown inlet plenum static pressure P0 at the upstream end of the heat sink and zero gauge pressure at the outlet.
- March downstream slice by slice: for each slice, solve for the local channel velocity Vch,i such that the pressure drop across the channels matches the local plenum pressure ($\Delta p \approx \frac{1}{2} \rho K_{ch,eff} V_{ch,i}^2$), compute the channel flow $q_i = A_{ch,slice} V_{ch,i}$, subtract qi from the remaining manifold flow, and update the plenum pressure via a Darcy loss in the manifold segment ($\Delta p_{pl,i} = f_{pl,i} (\Delta x / D_{h\_pl}) \frac{1}{2} \rho V_{pl,i}^2$).
- At a trial P0, the sum of all channel flows Σq_i generally does not equal the imposed Q*. A bisection loop on P0 is used until $\Sigma q_i \approx Q^*$ within a tolerance, at which point Vch(x) is recorded for plotting.

This provides a physically reasonable, but still low-order, estimate of how the channel velocity varies along the length of the heat sink as the plenum pressure decays.

4.6 3D Visualization and Velocity Coloring

The 3D view uses Three.js to render a simplified, scaled representation of the plenum, inlet, and heat sink:

- The plenum is a translucent rectangular box sized to the user-specified dimensions.
- The inlet is drawn as a solid block attached to the plenum at the inlet location.
- The heat sink base and representative fins are drawn with an aluminum-like color.
- Velocity-colored, semi-transparent blocks represent three regions: channels, top gap, and side gaps. Colors are assigned by ranking the branch velocities, with darker colors corresponding to higher velocities.
- A legend below the view reports the branch velocities in ft/s, consistent with the IP display of the module.

The 3D view is purely illustrative; it is not used directly in the solution algorithm but helps build intuition about flow paths and relative velocities.

5. Limitations and Recommended Use

The module is designed as a fast, engineering-level tool rather than a detailed CFD solver. Key assumptions and limitations include:

- Steady, incompressible flow of air only; compressibility and multiphase effects are neglected.
- Darcy friction correlations are simplified and do not capture very low-Re entrance effects or fully rough regimes.
- Minor loss coefficients are hard-coded as representative values (order unity) and are not tuned to a specific geometry.

- Heat transfer (h, surface temperature, fin efficiency) is not modeled; only fluid mechanics and fan air power are considered.
- The 1D Vch(x) model assumes uniform properties across the span and neglects side-to-side variations or 3D recirculation.
- The 3D view is a schematic representation with arbitrary visual scaling and does not replace detailed CAD or CFD.

Within these limits, the module is appropriate for parametric studies, geometry comparisons, and early sizing of plenum and heat sink configurations. Users should be cautious when applying the model to extreme geometries (very small gaps, highly non-uniform inlet conditions, or very high Reynolds numbers) and should validate critical designs with more detailed analysis or test data.

6. References and Further Reading

The underlying correlations and concepts are standard in fluid mechanics and heat transfer. Useful references include:

- Crane Co., "Flow of Fluids Through Valves, Fittings, and Pipe" (Technical Paper 410) – standard reference for Darcy–Weisbach equation, friction factors, and minor loss coefficients.
- Idelchik, I. E., "Handbook of Hydraulic Resistance" – comprehensive collection of pressure-loss data and correlations for various duct and passage geometries.
- Munson, Young, and Okiishi, "Fundamentals of Fluid Mechanics" (or equivalent fluid mechanics texts) – background on Reynolds number, laminar and turbulent flow, and the Blasius friction factor correlation.
- Shah, R. K., and London, A. L., "Laminar Flow Forced Convection in Ducts" – detailed treatment of laminar flow and heat transfer in channels and parallel-plate passages (for more advanced fin-channel modeling).
- ASHRAE Handbook – HVAC Systems and Equipment, fan manufacturers' application notes – conceptual discussion of fan and system curves, operating point, and fan selection methodologies.

These references provide the theoretical basis for the simplified correlations used in the module and can be consulted for more rigorous design or for tailoring minor loss coefficients to a specific hardware configuration.