Vertical Plate Natural Convection with Embedded Heat Sources and Fin Regions

This module estimates steady-state temperature distributions on a vertical rectangular plate with multiple embedded heat sources and optional finned regions exposed to air. The goal is to provide a quick, engineering-level tool for assessing the combined effects of conduction in the plate, natural convection to the ambient air, and added surface area from fins, without requiring a full CFD model.

1. How to Use the Module

1.1 Geometry and Material Setup

- Specify the plate width and height (m). The plate is assumed to be vertically oriented with natural convection acting on one exposed face.
- Specify the plate thickness (m) and select the plate material (Aluminum 6061, Copper, Steel, or FR4). The material database supplies thermal conductivity k, density ρ, and specific heat c_p; only k is used in the steady-state solution.
- Set the ambient air temperature (°C). This serves as the reference temperature for natural convection.
- Choose the number of finite-volume cells along the plate width (nx). The module automatically sets the number of cells along the height based on the plate aspect ratio so that the grid remains roughly square.

1.2 Defining Heat Sources

- Select the “Heat sources” draw mode.
- Use the mouse to drag rectangles on the plate canvas. Each rectangle defines a distributed heat source region.
- For each source S_1, S_2, …, enter its total power W in the sources panel. During assembly, the total power is divided equally among all cells covered by that source rectangle, yielding a uniform volumetric (per area) heat input over that region.
- The “Default power (W)” value is used for new source regions when they are first created.
- Heat source regions are drawn as semi-transparent red overlays on top of the temperature contour field.

1.3 Defining Fin Regions

- Select the “Fin regions” draw mode.
- Drag rectangles over the plate surface where fins are to be applied. Each region F_1, F_2, … is associated with a strip of equally spaced, straight, vertical fins mounted normal to the plate surface.
- For each fin region, specify:
  – Fin height H (m): distance from the plate surface to the fin tip.
  – Fin thickness t (m): fin thickness in the direction normal to the fluid flow between fins.
  – Fin spacing s (m): center-to-center pitch between fins, measured along the plate width.
  – Fin material: same as plate or one of the available materials. The fin conductivity k_f is used in the fin efficiency calculation.
- The module draws a end view schematic of the currently selected fin region below the main canvas. This view is scaled to the full plate width and shows the plate thickness and the fin geometry (H, t, s) over the selected region. The finned width in this detail view is consistent with the fin region width on the main canvas.
- Fin regions are displayed as semi-transparent blue overlays on the temperature field, with thin outlines for visual clarity.

1.4 Solving and Interpreting Results

- Press “Setup Grid” to rebuild the finite-volume mesh if any geometric input has changed.
- Press “Solve” to compute the steady-state plate temperature field with the current sources and fin regions.
- The legend at the upper right of the canvas shows the temperature range (Tmin and Tmax) mapped to the color bar (blue → green → yellow → red).
- A running status line above the input panel reports:
  – Grid resolution (nx × ny) and the current base convection coefficient h_base.
  – Plate area, total heater power ΣW, number of fin regions, approximate energy residual, and global average and maximum temperature.
- To probe local temperatures, click on the plate canvas. The module performs bilinear interpolation of the finite-volume solution to the cursor location and reports x, y and T in SI units directly under the canvas (the “Probe” line).
- “Reset to Ambient” restores the entire plate to the ambient temperature without changing the heaters or fins.

2. Solution Methodology

2.1 Governing Equations and Assumptions

The plate is treated as a two-dimensional conduction domain of thickness t, with steady-state energy balance in each finite volume:

$\nabla\cdot(k\,t\,\nabla T) + \dot{q}'' = h_{eff}\,(T_a - T)$

where k is the plate thermal conductivity, T is the local plate temperature, T_a is the ambient air temperature, $\dot{q}''$ is the net heat input per unit planform area from embedded sources, and h_{eff} is the effective local heat transfer coefficient to the ambient (baseline natural convection plus any added fin effect). The model assumes:

- Steady state (no transient storage term $\rho\, c_p\, t\, \partial T/\partial t$).
- Constant thermal properties k, ρ, c_p for each material.
- One-dimensional through-thickness temperature variation is neglected; the plate temperature is represented by a single T(x,y) field.
- Natural convection occurs on a single, unobstructed vertical face; radiation is neglected.
- Fins act by increasing the effective convection area (via fin efficiency and channel correlations), but the fins themselves are not spatially resolved with separate solid nodes; their effect is lumped into h_{eff} in the cells where they are present.

2.2 Spatial Discretization (Finite Volume Scheme)

The plate planform is discretized on a structured, uniform Cartesian grid with nx cells along the width and ny cells along the height. Let Δx and Δy be the cell dimensions, and T_{ij} the cell-centered temperature.

For each interior cell, the steady-state energy balance in finite-volume form is:

$a_E T_E + a_W T_W + a_N T_N + a_S T_S - a_P T_P = -Q_P$

where:

- $aE = aW = k\,t / (\Delta x^2)$
- $aN = aS = k\,t / (\Delta y^2)$
- $aP = aE + aW + aN + aS + h_{eff}$
- $QP = \dot{q}''P\,(\Delta x\,\Delta y) + h_{eff}\,Ta$

Here, h_{eff} is the effective local convection coefficient for that cell, and $\dot{q}''P$ is the heat-source power per unit area in that cell. Boundary cells use a simple mirroring approach (ghost values equal to the interior cell) so that conduction is zero normal to the plate boundary except where convection is applied.

2.3 Iterative Solver (SOR)

The discrete algebraic system is solved using successive over-relaxation (SOR). For each iteration k and cell P:

$TP^{(k+1)} = TP^{(k)} + \omega\,(TP^* - TP^{(k)})$

where TP^* is the value obtained by direct substitution of neighbor temperatures in the finite-volume balance, and ω is a relaxation factor (ω ≈ 1.8–1.9 in this implementation). The iteration proceeds cell-by-cell in line ordering until the maximum change in T over all cells falls below a specified tolerance.

2.4 Natural Convection Correlation for the Bare Plate

For the base case without fins, the effective convection coefficient is estimated using the Churchill–Chu correlation for laminar–turbulent natural convection from an isothermal, vertical plate over a wide range of Rayleigh number:

$$Ra_L = g\,\beta\,(\bar{T} - T_a)\,L^3 / (\nu\,\alpha)$$
$$Nu_L = 0.68 + [0.670\,Ra_L^{1/4}] / [\,(1 + (0.492 / Pr)^{9/16})^{4/9}\,]$$
$$h_{base} = Nu_L\,k_{air} / L$$

where L is the plate height, $\bar{T}$ is the current average plate temperature, β is the thermal expansion coefficient of air, ν is kinematic viscosity, α is thermal diffusivity, and Pr is the Prandtl number. In the code, air properties are treated as constants evaluated at a representative film temperature.

The module uses a Picard iteration on hbase: after each SOR sweep, the current average temperature $\bar{T}$ is used to update hbase via Churchill–Chu, and the process is repeated until hbase converges.

2.5 Fin Regions: Channel Model with Fin Efficiency

In finned regions, the effective convection coefficient is increased by two effects:

1. Enhanced natural convection within the vertical channels between parallel fins.
2. Added fin surface area, moderated by fin efficiency.

For each fin region F, with fin height Hf, thickness tf, spacing s, and fin conductivity kf, an overall increment ΔhF is computed and added to the local hbase for each covered cell.

Channel natural convection (Elenbaas / Bar-Cohen style):

The fin region is treated locally as a set of vertical channels of spacing S = s and height L (plate height). A Rayleigh number based on the spacing is formed:

$$Ra_S = g\,\beta\,(\Delta T)\,S^3 / (\nu\,\alpha)$$

An “Elenbaas-type” fully developed limit is estimated via

$Nu_{fd} \approx (Ra_S / 24) (S / L)$
$h_{fd} = Nu_{fd}\, k_{air} / S$

At the other extreme, if the spacing is large, each fin behaves more like an isolated vertical plate; a Churchill–Chu isolated-plate coefficient h_{iso} is formed based on Ra_L.
A blending function is then applied between these two limits using a mixed parameter $X \approx Ra_S (S / L)$, so that the effective channel coefficient h_{ch} transitions smoothly from h_{fd} at small spacing to h_{iso} at large spacing:

$$h_{ch} = (1 - w)\, h_{fd} + w\, h_{iso},$$

where w is a function of $\log_{10}X$ chosen to ramp from 0 to 1 over roughly one decade in X. This captures the expected loss of fin effectiveness at both extremely small spacing (high viscous resistance) and very large spacing (weak channeling and more isolated-plate behavior).

Fin efficiency and augmented area:

Each fin is modeled as a straight rectangular fin with convection coefficient h_{ch} on both sides and a base at the plate temperature. Using standard 1D fin theory with tip convection, a fin parameter m and efficiency η are defined:

$$m = \sqrt{2\, h_{ch} / (k_f\, t_f)}$$
$$\eta = \tanh(m H_f) / (m H_f)$$

Over a pitch $S + t_f$, the total external area per unit base width is approximately $A_{ext} \approx (S + t_f) + 2 H_f$, while the bare plate has area $(S + t_f)$. The added effective area ratio due to fins is then

$$R_A \approx 1 + \eta\, (2 H_f / (S + t_f)).$$

The additional heat transfer coefficient increment in a fin region is modeled as

$$\Delta h_F \approx h_{ch}\, \eta\, (2 H_f / (S + t_f)),$$

and this ΔhF is added to hbase for each cell within the fin region. This formulation ensures that as spacing becomes very large with weak channel interaction, hch → hiso and fins behave similar to isolated plates; as spacing becomes very small, hfd and the area ratio together limit the benefit, reflecting the increasing hydraulic resistance and loss of buoyancy-driven flow.

2.6 Energy Residual and Diagnostics

The module computes a simple global energy residual to check the balance between total input power and total convective removal:

Qin = Σ Wi (sum of all heater powers)
Qout = Σ heff P Acell max(TP − Ta, 0)
residual = |Qin − Qout| / (|Qin| + |Qout|)

The residual is reported as a percentage on the status line. For well-behaved cases, the residual is typically a few percent or less, indicating reasonable consistency between conduction and convection modeling.

3. Limitations and Recommended Use

This module is intended as a quick, engineering-level design and sensitivity tool, not a detailed CFD or full 3D conjugate heat transfer solver. Key limitations include:

- 2D plate approximation: through-thickness gradients and 3D edge effects are neglected.
- Constant air properties: buoyancy and viscous effects are modeled with fixed property values, suitable for moderate temperature differences in air.
- Empirical natural convection correlations: the Churchill–Chu and channel correlations are approximate and based on canonical geometries; complex enclosure effects, cross-flow, or strong interactions with neighboring hardware are not captured.
- Lumped fin effects: fins are represented by an effective increase in heff, not explicitly meshed. Fin root temperature variations along the base are neglected; the base is assumed isothermal locally.
- No radiation: surface-to-ambient radiation is not included, which can be important at elevated temperatures.

Within these limits, the tool is well suited for comparing different heat source layouts, fin heights, fin spacing, and material choices on the same plate, and for obtaining order-ofmagnitude temperature and hotspot estimates prior to more detailed analysis.

4. References

1. Churchill, S. W., and H. H. S. Chu, "Correlating Equations for Laminar and Turbulent Free Convection From a Vertical Plate," International Journal of Heat and Mass Transfer, Vol. 18, No. 11, 1975, pp. 1323–1329.

2. Incropera, F. P., DeWitt, D. P., Bergman, T. L., and Lavine, A. S., Fundamentals of Heat and Mass Transfer, 6th ed., Wiley, 2007. See chapters on free convection from vertical surfaces and extended surfaces (fins).

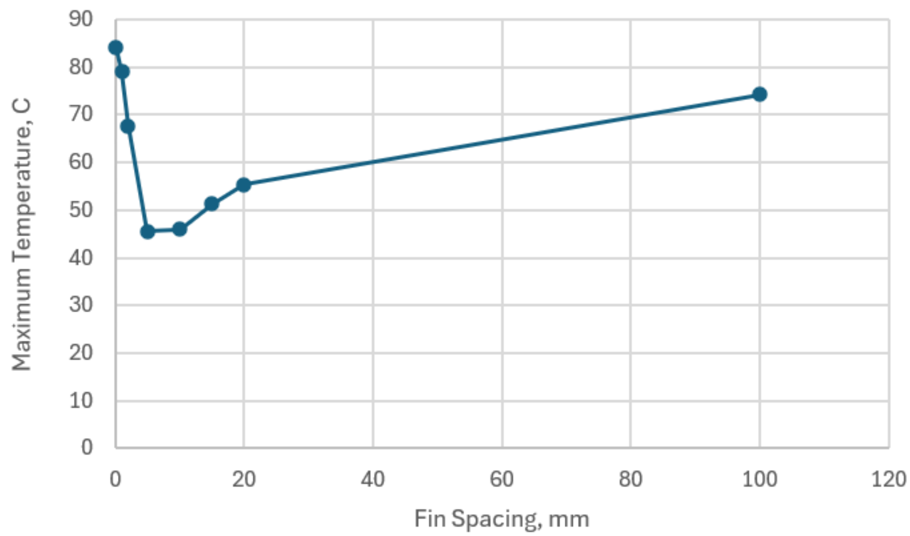
3. Bar-Cohen, A., and Rohsenow, W. M., "Thermally Optimum Spacing of Vertical Natural Convection Cooled Parallel Plates," ASME Journal of Heat Transfer, Vol. 106, 1984, pp. 116–123.

4. Elenbaas, W., "Heat Dissipation by Free Convection," Philips Research Reports, Vol. 3, 1948, pp. 1–24.

5. Validation

To test the model a case was set up to vary fin spacing. The plot shows the maximum temperature trend by varying the fin spacing, s, keeping the other parameter constant. For this case: plate width = .2m, height=.12m, aluminum, 1 heat source over the full plate area, finned over the full plate area with fin height=.02, fin thickness=.001.

As shown, there is an optimum spacing in the 6-12mm range as found by Bar-Cohen and Rohsenow of Reference 3. At the 2 extremes the temperature approaches the no-finned condition.



Maximum Temperature, C
90
80
70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
Fin Spacing, mm